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# FILM BOILING ON A HORIZONTAL SURFACE

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**Abstract**—An analysis of the heat-transfer process in the case of film boiling is made by taking into account the Taylor instability and the growth of the prominences up to the bubble departure. The initial radius of the prominences is determined by the Taylor instability, the final radius by the buoyant and surface tension forces. The growth of the prominences is due both to the Taylor instability and to a part of the vapour generated at the liquid–vapour interface by the heat flux. The rest of the vapour generated by the heat flux compensates the decrease of the film thickness caused by instability and maintains a stable film of vapour. The appendix contains equations for the diameter of the departing bubble which take into account also the friction and inertial forces.

## NOMENCLATURE

$A$, constant;
$a$, thermal diffusivity in the liquid;
$B$, constant;
$b$, growth coefficient, $b_D = b_{\lambda=\lambda_D}$;
$C$, drag coefficient;
$D_0$, equivalent diameter of the bubble which departs;
$g$, acceleration of gravitation;
$k$, thermal conductivity of vapour;
$q$, heat flux per unit area of plate;
$q'$, heat flux determined by means of equation (7);
$L$, latent heat of vaporization;
$l$, height of a prominence;
$m$, $2\pi/\lambda$;
$m'$, coefficient of apparent mass;
$p$, pressure;
$r$, distance to the centre of a prominence;
$R_c$, radius of cavity;
$R$, radius of a prominence;
$R_i$, initial value of $R$;
$R_f$, final value of $R$;
$\Delta T$, difference between the plate temperature and that of the liquid;
$T$, temperature in the bulk of the liquid [°K];
$t$, time;
$v$, velocity of vapour in motion, quasi-parallel to heating surface, within the film;
$x$, a distance along the plate.

Greek symbols

$\beta, \beta_0, \beta', \beta'', \beta'''$, constants;
$\gamma_v$, specific gravity of vapour;
$\gamma_l$, specific gravity of the liquid;
$\delta$, thickness of the film of vapour;
$\eta$, a distance representing the disturbance of the interface liquid–vapour;
$\lambda$, wavelength;
$\lambda_D$, dominant wavelength;
$\mu$, viscosity of vapour;
$\sigma$, surface tension.

In the case of film boiling on a horizontal surface, a quasi-steady film of vapour, from which bubbles depart quasi-periodically, separates the liquid from the heating surface. The problem of heat transfer in the case of film boiling has formed the object of several theoretical papers due to Chang [1], Zuber [2], Berenson [3] and Ruckenstein [4]. Chang was the first to point out that the film might exhibit waves whose wavelengths can be predicted from hydrodynamic considerations, Zuber derived an equation for the minimum heat flux, Berenson one for the heat-transfer coefficient, and the author equations for both quantities. The theories developed by Zuber, Berenson and Ruckenstein are based on the theory of instability of the interface between a heavier fluid situated over a lighter fluid; this instability is known under the name of Taylor instability [5–7]. In the Taylor instability theory small interface disturbances of the form

$$\eta = \eta_0 \, e^{bt} \cos mx \tag{1}$$

are considered, and if the effect of the thickness of the vapour film is neglected, the equation

$$b = \left[\frac{\gamma_l \gamma_v v^2 m^2}{(\gamma_l + \gamma_v)^2} + \frac{g(\gamma_l - \gamma_v)m}{\gamma_l + \gamma_v} - \frac{g\sigma m^3}{\gamma_l + \gamma_v}\right]^{\frac{1}{2}} \tag{2}$$

is obtained for the growth coefficient $b$.

For not too large values of the heat flux,

$$v^2 \ll \frac{g}{m} \frac{\gamma_l^2 - \gamma_v^2}{\gamma_l \gamma_v}$$

and equation (2) becomes

$$b = \left[\frac{g(\gamma_l - \gamma_v)m}{\gamma_l + \gamma_v} - \frac{g\sigma m^3}{\gamma_l + \gamma_v}\right]^{\frac{1}{2}}. \tag{3}$$

The interface is unstable for those perturbations for which $b > 0$, hence for perturbations whose wavelength is in the range

$$\lambda_C \equiv 2\pi\left(\frac{\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}} < \lambda < \infty$$

Zuber and Berenson consider that the boundary grows due to the instability (the growth rate being determined by the value of $b$) until it ruptures when a bubble departs from the node. There results that the distance between two neighbouring centres of bubble formation is at least equal to $\lambda_C$. Zuber considers this distance as lying between the critical value of the wavelength

$$\lambda_C = 2\pi\left(\frac{\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}} \tag{4}$$

and the dominant wavelength (the wavelength for which $b$ has a maximum)

$$\lambda_D = 2\pi\left(\frac{3\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}}, \tag{5}$$

while Berenson considers this distance equal to the dominant wavelength.

The experimental results obtained by Hosler and Westwater [8] seem to confirm the basic idea in the works of Zuber and Berenson that film boiling can be treated by means of Taylor's instability. Westwater observes among other things that the minimum distance between two bubble-generating points is equal to the critical wavelength and that the most frequent distance is equal to $\lambda_D$.

Using certain results from Taylor's theory of instability the author derived, in a paper published in 1962 [4], an equation for the heat flux. Unlike Berenson, who used in the examination of the heat-transfer process a stationary model of the process, in reference [4] a more realistic non-stationary model is suggested. The examination of the growth process of the bubbles allowed to point out, among others, a certain critical moment, corresponding to the minimum heat flux, and to derive an equation for it. Whereas the equation established by Zuber for the minimum heat flux was deduced as a limiting case of transition boiling, in reference [4] an equation, related in form, was derived as a limiting case of film boiling.

The aim of the present paper is to discuss the author's model and to compare it to that of Berenson.

## THE PHYSICAL MODEL

In the case of film boiling the rate of the process is determined by the heat transfer through a stable film of vapour separating the liquid from the heating surface. However, the interface between a heavier fluid situated over a lighter fluid is unstable for perturbations whose wavelength is in the range $\lambda_C < \lambda < \infty$. Since in any mechanical system there are perturbations of various wavelengths, including those in the mentioned range, the prominences of the interface created by instability increase rapidly and the vapour film would disappear if it were not maintained by the heat flux given to the system. The heat flux transferred through the vapour film existing between the prominences causes the vaporization uf the liquid at the interface. A part of the vapour produced compensates the decrease of the film in the space between prominences (this decrease is caused by the increase of perturbation) and the excess flows quasi-parallel with the heating surface towards the prominences created by perturbation. These prominences grow both due to instability and to the "excess" of vapour, until they become sufficiently large so that the buoyant force may equal the force due to surface tension, when bubbles break off. The perturbation created by the breaking off of the bubble produces a new prominence of the interface. This constitutes the nucleus whose development will lead to a new bubble.

The interface at the initial moment, the moment in which only the "nucleus" of the bubble exists, is illustrated in Fig. 1 (heavy line).

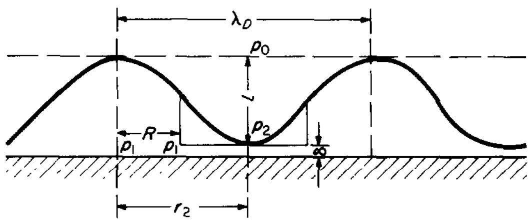


FIG. 1.

In order to simplify the computation the heavy curve is approximated in the interval between two "bubble nuclei" by the thinner line (Fig. 1). The thickness $\delta$ is considered practically independent of time and the heat flux transferred through the surface of the prominences is neglected in comparison with the heat flux transferred through the film of thickness $\delta$ in the space between the prominences. The distance between the centres of two successive prominences is taken equal to $\lambda_D$.

There is a critical moment when the vaporization produced at the interface, by the heat transferred to the liquid, compensates only the decrease of the film thickness. In this case, the prominences grow only due to the instability. This critical moment corresponds to the minimum heat flux, since if the heat flux were smaller than the one necessary for compensating the decrease of the thickness $\delta$, the vapour film could no longer subsist.

The model suggested in reference [4] is different from that used by Berenson in that the process of growth of the prominences from the initial radius (determined by the instability) to the final radius (determined by the buoyant force and the forces due to surface tension) is taken into account. In Berenson's model the final situation (the situation at the moment when the bubble departs) is considered as being realized all the time. The interaction between the effect of the instability and that of the heat flux seems to be better taken into account in the unsteady model.

**GENERATING RATE OF THE BUBBLES**

From the preceding section there results that the bubbles grow, on the one hand due to instability, and on the other one owing to a part of the vapour generated at the interface. Since we assume the thickness $\delta$ of the film of vapour to be practically constant, there results that the part of heat flux which produces a quantity of vapour which compensates the decrease of the film, contributes in an indirect manner to the increase of the prominences.

Let us divide the heating surface into squares of area $\lambda_D^2$, each square having in its centre a prominence. All the heat transferred through this surface to the liquid produces vapour which contributes to the growth of the prominence. A thermal balance leads to the equation

$$\beta_0 \frac{4}{3}\pi \frac{dR^3}{dt} = \frac{q'}{L\gamma_v}(\lambda_D^2 - \pi R^2), \tag{6}$$

in which owing to $\beta_0$ the left member of the equation represents the variation in time of the real volume of a prominence. Assuming laminar motion in the film, one may use for the heat flux $q'$ the expression

$$q' = \frac{k}{\delta}\Delta T. \tag{7}$$

Equation (6) can be written:

$$\mathfrak{R}^2 \frac{d\mathfrak{R}}{dt} = \frac{1}{\tau}(1 - \pi\mathfrak{R}^2), \tag{8}$$

where

$$\mathfrak{R} = \frac{R}{\lambda_D}$$

and

$$\tau = \frac{4\beta_0 \pi L \gamma_v \delta \lambda_D}{k\,\Delta T}.$$

By integrating the differential equation (8), the result

$$\mathfrak{R}_i - \mathfrak{R} + \frac{1}{2\pi^{\frac{1}{2}}} \ln \frac{(1 + \pi^{\frac{1}{2}}\mathfrak{R})(1 - \pi^{\frac{1}{2}}\mathfrak{R}_i)}{(1 + \pi^{\frac{1}{2}}\mathfrak{R}_i)(1 - \pi^{\frac{1}{2}}\mathfrak{R})} = \pi\frac{t}{\tau} \tag{9}$$

is obtained, where $\mathfrak{R}_i$ is the value of $\mathfrak{R}$ at the initial moment. The value of $R$ at the initial moment can be considered as dependent on the instability and therefore given by the expression

$$R_i = \beta'\lambda_D. \tag{10}$$

Since at the moment when the bubble departs the difference between the buoyant force and the weight of a bubble equals the surface tension force, there results*

$$R_f \propto D_0 = \beta\left(\frac{\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}}. \tag{11}$$

The time $\theta$ of formation of a prominence from which a bubble departs is given by the expression

$$\theta = \beta'''\tau, \tag{12}$$

where

$$\beta''' \equiv \frac{1}{\pi}(\mathfrak{R}_i - \mathfrak{R}_f) + \frac{1}{2\pi^{\frac{1}{2}}} \ln \frac{(1 + \pi^{\frac{1}{2}}\mathfrak{R}_f)(1 - \pi^{\frac{1}{2}}\mathfrak{R}_i)}{(1 + \pi^{\frac{1}{2}}\mathfrak{R}_i)(1 - \pi^{\frac{1}{2}}\mathfrak{R}_f)}.$$

The frequency of bubble generation is equal to $\theta^{-1}$.

## CALCULATION OF THE THICKNESS $\delta$ OF THE VAPOUR FILM

The calculation which follows is similar to a great extent to that performed by Berenson.

In order to simplify the calculation, the square of side $\lambda_D$ will be assimilated to an equivalent circle of radius $r_2 = \lambda_D \pi^{-\frac{1}{2}}$ and the hydrodynamic process in the film of thickness $\delta$ will be considered as having cylindrical symmetry.

For the average velocity of vapour moving, quasi-parallel with the heating surface, towards the prominence, in a section of height $\delta$ situated at a distance $r$ from the centre of the prominence, one can write the equation

$$L\gamma_v 2\pi r \delta v = \frac{k\,\Delta T}{\delta}(\lambda_D^2 - \pi r^2). \tag{13}$$

The average velocity $v$ can, on the other hand, be calculated, in quasi-stationary conditions, if one considers the motion laminar and neglects the inertial forces, by means of equation

$$\frac{dp}{dr} = A\frac{\mu v}{\delta^2}. \tag{14}$$

The constant $A$ is equal to 12 if the liquid–vapour boundary behaves, from the hydrodynamical point of view, like a solid surface and to 3 if it behaves like a free surface. From equations (13) and (14) there results

$$dp = \frac{A\mu k\,\Delta T}{L\gamma_v \delta^4}\,\frac{\lambda_D^2 - \pi r^2}{2\pi r}\,dr. \tag{15}$$

By integrating equation (15), one obtains

$$p_2 - p_1 = \frac{A\mu k\,\Delta T}{L\gamma_v \delta^4}\left[\frac{\lambda_D^2}{2\pi}\ln\frac{r_2}{R} - \frac{1}{4}(r_2^2 - R^2)\right]. \tag{16}$$

* A bubble breaks off when the difference between the buoyant force and the weight of the bubble equals the sum of surface forces, resistence forces and inertial forces. Evaluating the velocity in the expression of the resistence by $dR/dt$ (as suggested previously [9]) it may be shown that, in the vicinity of the minimum flux, equation (11) is valid even if the inertial and resistance forces are not negligible (see the Appendix for details).

However (see Fig. 1)

$$p_2 - p_0 = l\gamma_l$$

and if at the top of the prominence one considers the curvature radius equal to $R$,

$$p_1 - p_0 = l\gamma_v + \frac{2\sigma}{R}$$

and therefore

$$p_2 - p_1 = l(\gamma_l - \gamma_v) - \frac{2\sigma}{R} \tag{17}$$

Eliminating the pressure drop $p_2 - p_1$ between equations (16) and (17), one obtains for $\delta$ the equation

$$\delta^4 = \frac{A\mu k\,\Delta T}{\gamma_v L[l(\gamma_l - \gamma_v) - (2\sigma/R)]} \times \left[\frac{\lambda_D^2}{2\pi}\ln\frac{r_2}{R} - \frac{1}{4}(r_2^2 - R^2)\right]. \tag{18}$$

The thickness $\delta$ of the film depends, via $R$, on time. However, taking into account the fact that the initial value of $R$, $R_i$ and its final value $R_f$ are probably not very different from each other, and also the fact that the value of $\delta$ is given by the fourth root of the right-hand side of equation (18), the time dependence of $\delta$ is only slight.

Considering that

$$l \propto R.$$

and using for $R_i$ and $R_f$ expressions of the form

$$R_i \propto \left(\frac{\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}},$$

one obtains

$$\delta^4 \approx B\frac{\mu k\sigma^{\frac{1}{2}}\,\Delta T}{L\gamma_v(\gamma_l - \gamma_v)^{\frac{3}{2}}}. \tag{19}$$

This equation coincides with that established by Berenson in the framework of the stationary approximation.

The value of the proportionality constant $B$ in equation (19) depends on the value we choose for the proportionality constants which appear in the expressions of $l$, $R_i$ and $R_f$. Since this choice is to a certain extent arbitrary, we do not specify the value of the proportionality constant in equation (19).

The calculation proves that Berenson's stationary approximation leads to satisfactory results concerning the thickness of the vapour film.

### AN EQUATION FOR THE HEAT FLUX

From every surface of area $\lambda_D^2$ bubbles with an equivalent diameter (the diameter of a sphere having the same volume as the bubble) equal to $D_0$, are emitted at a rate equal to $1/\theta$. Therefore the heat flux $q$ can be expressed by means of the equation

$$q = L\gamma_v\frac{4\pi}{3}\left(\frac{D_0}{2}\right)^3\frac{1}{\theta\lambda_D^2}. \tag{20}$$

If for $D_0$ one uses equation (11) and for $\theta$ equation (12), one obtains

$$q = \frac{\beta^3}{576 \times 3^{\frac{1}{2}}\pi^2\beta_0 B^{\frac{3}{4}}}\,\frac{1}{\mathfrak{R}_i - \mathfrak{R}_f + \dfrac{1}{2\pi^{\frac{1}{2}}}\ln\dfrac{(1 + \pi^{\frac{1}{2}}\mathfrak{R}_f)(1 - \pi^{\frac{1}{2}}\mathfrak{R}_i)}{(1 - \pi^{\frac{1}{2}}\mathfrak{R}_f)(1 + \pi^{\frac{1}{2}}\mathfrak{R}_i)}}\left[\frac{\gamma_v L k^3(\Delta T)^3(\gamma_l - \gamma_v)^{\frac{3}{2}}}{\mu\sigma^{\frac{1}{2}}}\right]^{\frac{1}{4}} \equiv Z \times \left[\frac{\gamma_v L k^3(\Delta T)^3(\gamma_l - \gamma_v)^{\frac{3}{2}}}{\mu\sigma^{\frac{1}{2}}}\right]^{\frac{1}{4}}. \tag{21}$$

The value of the factor $Z$, which multiplies the square bracket, depends on the values of $\beta_0$, $\beta$, $B$, $\mathfrak{R}_i$ and $\mathfrak{R}_f$. For reasons specified in the preceding section, no choice of these values is made.

Equation (21) differs by the constant factor $Z$ from the corresponding equation established by

Berenson, this factor containing a series of characteristics of the unsteady model. This factor takes into account, among others, the fact that the surface which contributes to the process of heat transfer is larger at the initial moment than at the final one.

The experimental results obtained by Westwater are somewhat larger than those given by the equation derived by Berenson. A possible explanation of this difference might be found in the fact that in the stationary model used by Berenson only the final situation of the non-stationary model appears, a situation for which the real transfer surface is smaller than that corresponding to the initial moment.

### AN EQUATION FOR THE MINIMUM HEAT FLUX

It has been shown in the first part of the paper that for minimum heat flux the growth of the prominences is caused only by instability. For this reason the frequency of bubble emission will be calculated in this case by means of equation (1), assuming that the time $\theta$ is sufficiently short so that the growth of the perturbation may be calculated by means of the first order approximation.

Equation (1) leads to

$$R_f \approx R_i e^{b\theta}, \tag{22}$$

wherefrom

$$\theta \approx \frac{1}{b} \ln \frac{R_f}{R_i}.$$

However for $\lambda = \lambda_D$, equation (2) leads to

$$b_D = \frac{2^{\frac{1}{2}}}{3^{\frac{3}{4}}} \left[ \frac{g(\gamma_l - \gamma_v)^{\frac{3}{2}}}{(\gamma_l + \gamma_v)\,\sigma^{\frac{1}{2}}} \right]^{\frac{1}{2}}. \tag{23}$$

Therefore, under conditions of minimum heat flux

$$\theta = \frac{3^{\frac{3}{4}}}{2^{\frac{1}{2}}} \left( \ln \frac{R_f}{R_i} \right) \left[ \frac{(\gamma_l + \gamma_v)\,\sigma^{\frac{1}{2}}}{g(\gamma_l - \gamma_v)^{\frac{3}{2}}} \right]^{\frac{1}{2}}. \tag{24}$$

By using also equations (20 and (11), one obtains for the minimum heat flux $q_{\min}$ the equation

$$q_{\min} = \frac{2^{\frac{1}{2}}\beta^3}{72\pi \times 3^{\frac{3}{4}} \ln (R_f/R_i)} L\gamma_v \left[ \frac{\sigma g^2(\gamma_l - \gamma_v)}{(\gamma_l + \gamma_v)^2} \right]^{\frac{1}{4}}$$

$$\equiv Z_1\, L\gamma_v \left[ \frac{\sigma g^2(\gamma_l - \gamma_v)}{(\gamma_l + \gamma_v)^2} \right]^{\frac{1}{4}}. \tag{25}$$

Equation (25) differs from that established by Zuber by the constant factor $Z_1$, a factor depending on a number of characteristics of the unsteady model, namely on the values of $R_i$ and $R_f$.

The remark may be made that while the equation established by Zuber has been deduced as a limiting case of transition boiling by postulating that the generating frequency is determined by instability growth only, equation (25) has been derived as a natural limiting case of film boiling, without postulating this fact.

### ONSET OF FILM BOILING

Two comments concerning the onset of film boiling will be made in this section.

1. For the formation of a vapour film between the liquid and the heating surface, it is necessary that the number of active centres be large (so that the coalescence of the bubbles may take place) and that this film be stable from the hydrodynamical point of view.

The number of active centres necessary to form an unstable film through the coalescence of bubbles probably exists even for values of $\Delta T$ close to the value corresponding to the maximum flux. As an argument in favour of this statement one may quote the experimental findings of Westwater and Santangelo [10]. Indeed, the violent state of the vapour film in the range of transition boiling is probably the consequence of two opposite effects. On one hand, owing to the numerous active centres, there is a tendency of film formation and on the other hand, owing to its instability, the film breaks. At the points where the film breaks the liquid comes into contact with the heating surface. In these points there are certainly active centres on which bubbles are formed which push back the liquid*. These continuous breakings and re-formings of the film determine the state of violent motion that has been experimentally observed.

The existence of the transition boiling shows that the condition linked to the nucleating properties of the heating surface is already fulfilled long before film boiling appears. It is however possible, at least in principle, that this condition may not be fulfilled for values of $\Delta T$ larger than those required by the condition of the film stability. In those cases the film boiling occurs at that value of $\Delta T$ for which the condition of nucleation is also satisfied.

2. Katz [11] and Gaertner [12] found that the chemical nature of the heated surface may control the onset of film boiling. In the experiments of Gaertner double distilled water was boiled on two surfaces nonwetted by water; one of them coated with a thin film of polytetrafluoroethylene and the other with a thin film of a silicone grease. He found that "after bubbles formed at nucleation sites they did not rise into the bulk liquid, but rather grew and spread on the surface, coalesced with their neighbours, and soon generated a continuous blanket of vapour over the entire surface. Heat transfer by nucleate boiling was impossible".

This phenomenon is compatible with the hydrodynamic instability theory. Let us consider disturbances of the vapour–liquid interface equal to the thickness of the vapour film. If the liquid does not wet the solid surface, the regions in which the liquid comes into contact with the solid surface are soon recovered by vapour and the destruction of the film is hindered. The opposite effect occurs in the case in which the liquid wets the solid surface. Unfortunately the problem of the stability of the film under the combined action of large disturbances equal to the film thickness and that of the wetting forces can not be solved theoretically for the time being.

The Taylor instability theory does not and cannot take into account the effect of wetting of the solid surface, since it is a first order perturbation theory valid as long as the disturbances of the interface are small as compared to the film thickness. Nevertheless the treatment based on the Taylor instability theory constitutes probably a satisfactory approximation for partially wetted surface, the effect of the wetting forces being in such cases of secondary importance.

* The time $\tau'$ of contact between the liquid and the solid surface may be evaluated by means of the equation:

$$\frac{2\sigma}{R_c}\frac{T}{\gamma_v L} = (\Delta T)\,\mathrm{erfc}\,\frac{R_c}{2\sqrt{(a\tau')}}$$

established by the author [13] and later by Hsu [14] for the waiting period in the case of nucleate boiling.

## SUMMARY AND CONCLUSION

An analysis of the heat-transfer process in the case of film boiling is made by taking into account both the process of formation of the "nuclei of bubbles" and the growth of these prominences up to the bubble departure. The initial radius of these prominences is determined by instability, and the final radius (from the moment the bubble departs) by the buoyant and the surface tension forces. The prominences grow both due to the Taylor instability and to a part of the vapour generated at the liquid–vapour interface by the heat flux. The rest of the vapour generated by the heat flux compensates the decrease of the film thickness caused by instability and maintains a quasi-stable film of vapour. There is a critical moment for which the vaporization at the liquid–vapour interface compensates only the decrease of the film thickness. In this case the growth of the prominences is caused only by instability. This critical moment corresponds to the minimum flux since, if the heat flux were smaller than the one necessary for compensating the decrease of the film, the film of vapour could no longer subsist.

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## APPENDIX

### *Calculation of the Diameter of the Departing Bubble*

A bubble breaks off when the volume of the prominence becomes sufficiently large so that the difference between the buoyant force and the weight of the bubble equals the sum of the surface forces, resistence forces and inertial forces

$$(\gamma_l - \gamma_v)\frac{\pi}{6}D_0^3 = \frac{1}{2}\frac{\gamma_l}{g}C\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2_{R=R_f}\pi R_f^2 + \tfrac{4}{3}\pi\beta_0\frac{\gamma_v + m'\gamma_l}{g}\times\left(\frac{\mathrm{d}(R^3\,\mathrm{d}R/\mathrm{d}t)}{\mathrm{d}t}\right)_{R=R_f} + \beta''\pi D_0\sigma. \qquad \text{(A1)}$$

The first term on the right-hand side represents the resistence of the liquid; as suggested in a previous paper concerning the case of nucleate boiling [9], the velocity is evaluated by means of $\mathrm{d}R/\mathrm{d}t$. The second term on the right-hand side represents the inertial forces including via $m'$ the apparent mass. The last term on the right-hand side represents the surface forces.

At the minimum flux the growth of the prominences is due only to instability. For this reason one may use for $R$ equation (1). This equation leads to

$$\frac{\mathrm{d}R}{\mathrm{d}t} = b_D R$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(R^3\frac{\mathrm{d}R}{\mathrm{d}t}\right) = 4b_D^2R^4.$$

Equation (A1) becomes

$$(\gamma_l - \gamma_v)\frac{\pi}{6}D_0^3 = \left(\frac{\pi}{2}\frac{\gamma_l}{g}C + \frac{16}{3}\pi\beta_0\frac{\gamma_v + m'\gamma_l}{g}\right)b_D^2R_f^4 + \beta''\pi D_0\sigma. \qquad \text{(A2)}$$

If the resistance and the inertial terms are negligible as compared to the surface forces term, one obtains

$$R_f \propto \left(\frac{\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}}. \qquad \text{(A3)}$$

If the surface forces term is negligible,

$$R_f \propto \frac{g}{b_D^2}\frac{\gamma_l - \gamma_v}{(\pi/2)\gamma_l C + \frac{16}{3}\pi\beta_0(\gamma_v + m'\gamma_l)},$$

which, using equation (23) for $b_D$, becomes

$$R_f \propto \frac{\gamma_l + \gamma_v}{(\pi/2)\gamma_l C + \frac{16}{3}\pi\beta_0(\gamma_v + m'\gamma_l)}\left(\frac{\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}}. \qquad \text{(A4)}$$

The evaluations made show that the inertial term and the resistance term may be larger than the surface forces term. Nevertheless, the treatment of the minimum heat flux based on equation (A3) remains valid, since usually $\gamma_v \ll \gamma_l$ and in these cases equations (A3) and (A4) have the same form.

If all the terms of equation (A2) are of importance, it may be shown easily that

$$R_f \propto \left(\frac{\sigma}{\gamma_l - \gamma_v}\right)^{\frac{1}{2}} f\left(\frac{\gamma_v}{\gamma_l}\right). \quad \text{(A5)}$$

where function $f(\gamma_v/\gamma_l)$ = constant for $\gamma_v \ll \gamma_l$.*

For $q > q_{\min}$ the growth of the prominences is due not only to the instability, but also to the heat flux. In this case equation (A2) is no longer valid. The use of equation (6), allows to write equation (A1) under the form:

$$(\gamma_l - \gamma_v)\frac{\pi D_0^3}{6} = \frac{1}{2}\frac{\gamma_l}{g}C\left(\frac{q'}{4\beta_0\pi L\gamma_v}\right)^2 \times \left(\frac{\lambda_D^2 - \pi R_f^2}{R_f^2}\right)^2 \pi R_f^2 + \frac{1}{12\beta_0\pi}\frac{\gamma_v + m'\gamma_l}{g}\left(\frac{q'}{L\gamma_v}\right)^2 \times \frac{(\lambda_D^2 - 3\pi R_f^2)(\lambda_D^2 - \pi R_f^2)}{R_f^2} + \beta''\pi D_0\sigma. \quad \text{(A6)}$$

Since it is difficult to use such an equation for $D_0$, we shall use in the vicinity of the minimum flux the expressions valid for the minimum flux.

* We note that equation (A5) suggests for $q_{\min}$ a correlation of the type

$$\frac{q_{\min}}{L\gamma_v\left[\frac{\sigma g^2(\gamma_l - \gamma_v)}{(\gamma_l + \gamma_v)^2}\right]^{\frac{1}{4}}} = F\left(\frac{\gamma_v}{\gamma_l}\right).$$

**Résumé**—La théorie du processus du transport de chaleur dans l'ébullition par film est faite en tenant compte de l'instabilité de Taylor et de la croissance des protubérances jusqu'au départ des bulles. Le rayon initial des protubérances est déterminé par l'instabilité de Taylor et le rayon final par les forces d'Archimède et de tension superficielle. La croissance des protubérances est due à la fois à l'instabilité de Taylor et à une partie de la vapeur produite par le flux de chaleur à l'interface liquide–vapeur.

Le reste de la vapeur engendrée par le flux de chaleur compense la décroissance de l'épaisseur du film produite par l'instabilité et maintient un film de vapeur stable. L'annexe comprend les équations pour le diamètre de la bulle qui se détache en tenant compte également des forces de frottement et d'inertie.

**Zusammenfassung**—Für den Fall des Filmsiedens wird eine Analyse des Wärmeübergangsprozesses vorgenommen, wobei die Taylor-Instabilität und das Wachstum der Erhebungen der Wellenberge bis zum Ablösen der Blasen berücksichtigt werden. Der anfängliche Radius der Erhebung wird mit Hilfe der Taylor-Instabilität, der Ablöseradius mit Hilfe der Auftriebs- und Oberflächenspannungskräfte bestimmt. Das Wachstum der Erhebungen wird sowohl von der Taylor-Instabilität als auch von einem Teil des Dampfes bestimmt, der auf Grund des Wärmestromes an der Phasengrenze Dampf–Flüssigkeit entsteht. Der Rest des Dampfes, der sich auf Grund des Wärmestromes bildet, kompensiert die von der Instabilität herrührende Abnahme der Filmdicke, und bedingt einen stabilen Dampffilm.

Der Anhang enthält Fragen des Blasenablösedurchmessers bei Berücksichtigung der Reibungs- und Trägheitskräfte.

**Аннотация**—Проведен анализ процесса теплообмена при пленочном кипении с учетом нестационарности Тейлора и роста выпуклостей до отрыва пузырьков. Начальный радиус выпуклости определяется нестационарностью Тейлора, а конечный — выталкивающей силой и силой поверхностного натяжения. Рост пузырьков происходит благодаря нестационарности Тейлора, и частично за счет парообразования на поверхности раздела жидкость-пар из-за теплового потока. Остальной пар, образованный за счет теплового потока, компенсирует уменьшение толщины пленки, обязанное нестационарности, и поддерживает устойчивую пленку пара. В приложении приводятся уравнения для диаметра отрывающегося пузырька с учетом сил трения и инерции.